Amortization Modeling of Sustainability-Linked Loans in the Renewable Energy Sector using Linear Non-Homogenous Recurrence Relations

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Abstract**—*****The rapid expansion of renewable energy financing has driven the adoption of Sustainability-Linked Loans (SLLs), a class of loan instruments whose interest rates are adjusted dynamically based on the borrower's achievement of predetermined Key Performance Indicators (KPIs). Unlike conventional fixed-rate loans, the piecewise-varying rate structure of SLLs renders standard closed-form amortization formulas insufficient. This paper proposes a mathematical framework for modeling SLL amortization using first-order linear non-homogeneous recurrence relations, deriving a closed-form expression for the outstanding principal balance B_n at any period n within a given KPI milestone interval. The general solution is obtained by combining the homogeneous solution with a constant particular solution, and is chained sequentially across milestone intervals to produce the complete repayment path/trajectory. A numerical simulation is conducted on a hypothetical USD 10 million, 60-period wind farm financing case, comparing three KPI outcome scenarios: full KPI achievement (step-down rate), baseline (constant rate), and full KPI failure (step-up rate). Results demonstrate that consistent KPI achievement reduces total interest paid by approximately 11.8% relative to the baseline, while consistent underperformance increases the interest burden by 12.8% and leaves an unpaid residual balance of $247,691.53 at loan maturity. These findings confirm that the recurrence relation framework provides a rigorous and transparent computational tool for evaluating the financial implications of sustainability performance in green lending.*****

Keywords**—*****amortization; sustainability-linked loans; renewable energy; recurrence relations; discrete mathematics*****

I. INTRODUCTION

The transition towards clean and sustainable energy has become a defining economic and environmental priority in this current era. Data from the International Renewable Energy Agency (IRENA) showed that renewable energy capacity additions reached a record of 692 gigawatts (GW) in 2025, driven by the expansion of solar, wind, and hydropower installations across both developed and developing nations [1]. This momentum is further driven by international commitments such as the Paris Agreement and the United Nations Sustainable Development Goal 7 (UN SDG 7) which collectively push governments and private sectors to lower the implications of climate change by decarbonizing their energy systems by 2050 [2]. However, fulfilling these targets requires an unprecedented amount of capital. The International Energy Agency estimates that the world needs to invest a sum of $4.5 trillion annually through 2030 in order to meet net-zero emission targets [3].

One financial instrument that has gained spotlight are Sustainability-Linked Loans (SLLs). By formal definition, SLLs are types of loan instruments for which the financial/structural characteristics (especially interest rates) vary depending on the borrower's performance; i.e., whether the borrower achieves quantifiable predetermined sustainability performance objectives [4]. Hence, unlike conventional loans, SLLs are structured around the borrower's sustainability performance. The interest rate of SLL is tied to the achievement of predetermined Key Performance Indicators (KPIs). If the borrower meets or exceeds these KPI thresholds at agreed milestone periods, the interest rate is adjusted downward. Conversely, if the borrower fails to meet the KPIs, the interest rate is adjusted upwards, therefore granting financial penalties to projects which fail to meet its sustainability commitments.

Despite the growing adoption of SLLs in the renewable energy sector, the mathematical modeling of their repayment structure remains largely underexplored. Conventional loan amortization models are built upon the assumption that the interest rate remains constant throughout the entire loan tenor. Hence, under this assumption, the outstanding balance at each period follows a straightforward geometric progression, and the repayment schedule can be easily derived in a closed form. However, this assumption does not apply in the context of SLLs, as the interest rate is not static but instead subject to discrete adjustments at predefined KPI milestone periods. Therefore, since the interest rate changes over time, the balance recurrence relation includes a non-constant term, making standard closed-form amortization formulas inapplicable in this case. This current gap motivates the need for a more principled mathematical approach to SLL amortization modeling.

This paper proposes the use of linear non-homogenous recurrence relations to model the amortization dynamics of SLLs in the renewable energy sector. A recurrence relation expresses the value of a sequence at a given period as a function of its values at one or more preceding periods. In the context of standard loan amortization, the outstanding principal balance at period n, denoted B_n is recursively defined in terms of the balance at the previous period B_{n-1}, the applicable interest rate r_n, and the fixed periodic installment P, yielding the relation $B_n = (1 + r_n)B_{n-1} - P$. When r_n is constant, this reduces to a first-order linear recurrence relation with a constant forcing term, resulting in a well-known closed-form solution. However, when r_n varies across milestone periods as it is with SLLs, the recurrence relation becomes non-homogeneous, requiring the application of more general solution techniques such as the method of particular solutions combined with the homogenous solution.

II. THEORETICAL FRAMEWORK

A. *Sustainability-Linked Loans*

Sustainability-linked loans refers to financial agreements where a loan's terms, especially its interest rate, fluctuate based on the borrower's ability to meet specific sustainability goals. These goals, commonly referred to as Sustainability Performance Targets (SPTs) or Key Performance Indicators (KPIs), are agreed upon between the borrower and lender prior to loan origination and are assessed at discrete milestone periods throughout the loan tenor. The Loan Market Association and the Loan Syndications and Trading Association has outlined three core components that characterize a valid SLL structure in its SLL Principles guidebook [4]: the selection of ambitious yet relevant KPIs, the calibration of appropriately stretching KPIs, and the provision of transparent reporting and verification mechanisms.

In the renewable energy sector, some examples of the KPIs include the percentage of total energy consumption sourced from renewable sources, the total installed capacity of clean energy infrastructure, and the reduction in greenhouse gas (GHG) emissions relative to a baseline year. At each milestone period (generally occurs annually or semi-annually), the borrower's performance against the agreed KPIs are assessed. If performance meets or exceeds the target, the loan's interest rate for the subsequent period is reduced by a predetermined margin. Borrowers who fail to reach the agreed KPIs result in a rate increase for the loan.

B. *Amortization Theory*

Loan amortization refers to the process of repaying a loan through a series of regular payments that gradually reduce both the principal and interest, ultimately paying off the debt over a set period [5]. In the standard annuity amortization model, a borrower repays a fixed amount P at the end of each period over a total tenor of N periods. The outstanding principal balance at the beginning of period n, denoted B_n evolves as follows.

- The interest accrued during period n is $r \cdot B_{n-1}$, where r is the periodic interest rate.
- The principal repaid during period n is $P - r \cdot B_{n-1}$
- The updated balance is therefore $B_n = (1 + r)B_{n-1} - P$.

Given an initial loan principal $B_0 = L$, the closed-form expression for B_n under a constant rate r is:

$$B_n = L(1 + r)^n - P\frac{(1+r)^n-1}{r}$$

The fixed installment P that ensures full repayment by period N is given by setting $B_N = 0$, such that:

$$P = L\frac{r(1+r)^N}{(1+r)^N-1}$$

C. *Recurrence Relations*

A recurrence relation for a sequence $\{a_n\}$ is an equation that establishes the value of the n-th term, a_n, by using one or more of its preceding terms (a_0, a_1, a_2, …, a_{n-1}). This relationship applies to all integers $n \geq n_0$, where the base index n_0 is a non-negative integer. Any sequence whose elements fulfill the conditions of this equation is recognized as a solution to the recurrence relation [6]. Recurrence relations arise naturally in a wide range of applications in mathematics, computer science, and finance, including counting/combinatorial problems, algorithms analysis, and compound interest as well as loans.

A linear recurrence relation of degree k with constant coefficients takes the general form as follows.

$$a_n = c_1a_{n-1} + c_2a_{n-2} + \ldots + c_ka_{n-k} + f(n)$$

where c_1, c_2, ..., c_k are real constants and $f(n)$ is a function of n. When $f(n) = 0$ for all n, the relation is called homogenous; otherwise, it is called non-homogenous, with $f(n)$ referred to as the non-homogenous term.

The general solution to a linear non-homogenous recurrence relation is composed of two parts:

$$a_n = a_n^{(h)} + a_n^{(p)}$$

where $a_n^{(h)}$ is the homogenous solution (the general solution to the associated homogenous relation obtained by setting $f(n) = 0$), and $a_n^{(p)}$ is a particular solution that satisfies the full non-homogenous relation.

For a first-order linear non-homogenous recurrence relation of the form:

$$a_n = c \cdot a_{n-1} + f(n)$$

The homogenous solution is $a_n^{(h)} = Ac^n$, where A is a constant determined by the initial state. When $f(n) = d$ such that d is a nonzero constant, and c is not assumed to be 1, the particular solution takes the form of $a_n^{(p)} = \frac{d}{1-c}$, which results in the general solution:

$$a_n = Ac^n + \frac{d}{1-c}$$

III. Mathematical Modeling

A. Problem Formulation

Consider a Sustainability-Linked Loan with an initial principal $B_0 = L$, a fixed periodic installment P, and a total tenor of N periods. The loan tenor is partitioned into M milestone intervals, where each interval $[n_{m-1}, n_m)$ corresponds to a period during which a constant interest rate r_m applies, determined by the borrower's KPI performance evaluated at the beginning of that interval. Formally, the interest rate function is defined piecewise as:

$$r_n = r_m, \text{ for } n_{m-1} \le n < n_m, m = 1, 2, \ldots, M$$

The rate r_m is determined according to the following KPI-based adjustment rule:

$$r_m = \begin{cases} r_{\text{base}} - \delta & \text{if KPI target is met at milestone } m \\ r_{\text{base}} & \text{if KPI target is not assessed at milestone } m \\ r_{\text{base}} + \delta & \text{if KPI target is not met at milestone } m \end{cases}$$

where r_{base} denotes the base interest agreed upon at the beginning of the loan period and $\delta > 0$ denoting the rate adjustment margin. The main problem is therefore to derive a closed-form expression for the outstanding balance B_n at any period n within the loan tenor, under this rate structure similar to a piecewise function.

Since B_n is defined recursively in terms of B_{n-1} by a rate that varies by milestone interval, this problem could be naturally formulated as a system of linear non-homogenous recurrence relations, one for each milestone interval.

B. Recurrence Relation for Sustainability-Linked Loan Amortization

The outstanding balance for any given milestone interval where the rate r_m applies is as follows:

$$B_n = (1 + r_m)B_{n-1} - P, \quad n_{m-1} < n \le n_m \tag{1}$$

This is considered as a first-order linear non-homogenous recurrence relation of the form $a_n = c \cdot a_{n-1} + f(n)$, where $c = (1 + r_m)$ and $f(n) = -P$ as the constant non-homogenous term.

To first find the homogenous solution, P shall be set to 0, and the associated homogenous relation is:

$$B_n^{(h)} = (1 + r_m)B_{n-1}^{(h)}$$

whose solution is:

$$B_n^{(h)} = A_m(1 + r_m)^n$$

Such that A_m is a constant to be determined by the initial condition of the m-th interval. Meanwhile, to find the particular solution, it is assumed that a constant particular solution $B_n^{(p)} = K$ exists, since $f(n) = -P$ is a nonzero constant and $(1 + r_m) \ne 1$ as $r_m > 0$. By substituting into equation (1), it is found that:

$$K = (1 + r_m)K - P$$

$$K - (1 + r_m)K = -P$$

$$K = \frac{P}{r_m}$$

To find the general solution, the homogenous and particular solutions are combined.

$$B_n = A_m \cdot (1 + r_m)^n + \frac{P}{r_m} \tag{2}$$

Finally, to determine the constant A_m, the initial condition at the start of interval m is used, namely $B_{n_{m-1}}$ which represents the outstanding balance carried over from the previous interval. Substituting $n = n_{m-1}$ to equation (2), A_m could be solved then.

$$B_{n_{m-1}} = A_m(1 + r_m)^{n_{m-1}} + \frac{P}{r_m}$$

$$A_m = (B_{n_{m-1}} - \frac{P}{r_m})(1 + r_m)^{-n_{m-1}} \tag{3}$$

Substituting equation (3) back to equation (2) results in the closed-form expression for B_n within interval m.

$$B_n = (B_{n_{m-1}} - \frac{P}{r_m})(1 + r_m)^{n - n_{m-1}} + \frac{P}{r_m} \tag{4}$$

for $n_{m-1} \le n \le n_m$. This expression gives the exact outstanding balance at any period n within the m-th milestone interval, given the balance at the start of that interval and the applicable rate r_m.

C. Closed-Form Solution Across All Intervals

The complete amortization form of the SLL is obtained by applying equation (4) across all M milestone intervals, using the final balance of each interval as the initial condition for the next. The balance at the end of interval m serves as the initial condition for interval $m + 1$:

$$B_{n_m} = (B_{n_{m-1}} - \frac{P}{r_m})(1 + r_m)^{n_m - n_{m-1}} + \frac{P}{r_m} \qquad (5)$$

To find the full outstanding balance path of $\{B_0, B_1, B_2, ..., B_N\}$, it is determined by the following.

1. The initial principal $B_0 = L$
2. The fixed installment P, computed based on the base rate r_{base} as follows.

$$P = L\frac{r_{base}(1+r_{base})^N}{(1+r_{base})^N - 1}$$

3. The piecewise interest rate based on KPI outcomes, $\{r_1, r_2, ..., r_M\}$
4. The milestone period boundaries, $\{n_0, n_1, n_2, ..., n_M\}$

IV. NUMERICAL EXPERIMENT

A. *Setup and Assumptions*

In order to demonstrate the applicability of the mathematical model derived in the previous chapter, a numerical simulation is conducted based on a hypothetical situation of a bank issuing a Sustainability-Linked Loan to finance a utility-scale wind farm project. The loan parameters are as of the following.

Financial Metric	**Nominal**
Initial principal	$L = \$10,000,000$
Loan tenor	$N = 60$ periods (5 years with monthly installments)
Base interest rate	$r_{base} = 0.6\%$ per month (approximately 7.2% per year)
Rate adjustment margin	$\delta = 0.05\%$ per month
Number of KPI milestone intervals	$M = 5$
Milestone boundaries	$n_0 = 0, n_1 = 12, n_2 = 24,$ $n_3 = 36, n_4 = 48,$ $n_5 = 60$
KPI metric	Percentage of installed renewable capacity compared relatively to the project target

The fixed periodic installment P shall be computed as follows based on r_{base} as follows.

$$P = 10000000 \cdot \frac{0.006(1.006)^{60}}{(1.006)^{60} - 1}$$

$$P \approx \$198,956.85 \text{ per month}$$

This installment would remain fixed throughout the entire loan tenor regardless of KPI outcomes. What varies across scenario is the applicable rate r_m within each milestone interval, which in turn affects the interest rate of each payment and the path/trajectory of the outstanding balance B_n.

There are assumed three scenarios which are considered to reflect different KPI outcomes.

1. Scenario A (Full Achievement): The borrower achieves all KPI targets at every milestone, resulting in a step-down rate adjustment of -δ applied at each milestone period.
2. Scenario B (Control Group/No Adjustment): Set as a control group to be compared towards scenario A and C. The borrower's KPI performance is assumed to not be assessed, and hence the rate stays constant at $r_{base} = 0.60\%$.
3. Scenario C (Total Failure): The borrower fails to meet all KPI targets at every milestone, resulting in a step-up rate adjustment of +δ at every milestone.

The rate sequence can be seen as follows.

Scenario	**Rate Sequence**
A	{0.60%, 0.55%, 0.50%, 0.45%, 0.40%}
B	{0.60%, 0.60%, 0.60%, 0.60%, 0.60%}
C	{0.60%, 0.65%, 0.70%, 0.75%, 0.80%}

B. *Manual Computation of Outstanding Balance*

As an illustration, the computation within the first two milestone intervals of Scenario A is shown explicitly below.

Interval 1 ($n_0 = 0$ to $n_1 = 12, r_1 = 0.006$):

Initial condition: $B_0 = 10,000,000$

$$B_n = \left(10,000,000 - \frac{198,939.15}{0.006}\right)(1.006)^n + \frac{198,939.15}{0.006}$$

$$B_n = (10,000,000 - 33,156,525.00)(1.006)^n + 33,156,525.00$$

$$B_n = -23,156,525.00 \cdot (1.006)^n + 33,156,525.00$$

$$B_{12} = -23,156,525.00 \cdot (1.006)^{12} + 33,156,525.00$$

$$B_{12} \approx -23,156,525.00 \cdot 1.074424 + 33,156,525.00$$

$$B_{12} \approx \$8,276,464.29$$

Interval 2 ($n_1 = 12$ to $n_2 = 24$, $r_2 = 0.0055$; reduced by 0.05%):

Initial condition: B_{12} = 8,276,464.29

$$B_n = \left(8,276,464.29 - \frac{198,939.15}{0.0055}\right)(1.0055)^{n-12} + \frac{198,939.15}{0.0055}$$
$$B_n = (8,276,464.29 - 36,170,754.55)(1.0055)^{n-12} + 36,170,754.55$$
$$B_n = -27,894,290.26 \cdot (1.0055)^{n-12} + 36,170,754.55$$
$$B_{24} = -27,894,290.26 \cdot (1.0055)^{24-12} + 36,170,754.55$$
$$B_{24} \approx -27,894,290.26 \cdot 1.068004 + 36,170,754.55$$
$$B_{24} \approx \$6,379,161.42$$

By the use of Excel, the same procedure is applied for all remaining intervals and for scenario B and C. The full results are summarized in the table below (slightly rounded).

Period	Scenario A	Scenario B	Scenario C
0	$10,000,000	$10,000,000	$10,000,000
12	$8,276,374.13	$8,276,374.13	$8,276,374.13
24	$6,378,399.98	$6,424,468.84	$6,470,804.87
36	$4,317,559.89	$4,434,737.04	$4,554,194.72
48	$2,109,097.00	$2,296,921.11	$2,492,938.38
60	−$228,123.86	$0.00	$247,691.53

In scenario A, the interest paid is reduced by $228,123.86 as a consequence of the lowered interest rates. However, in scenario C, the interest paid is substantially increased by $247,691.53 due to higher interest rates.

C. *Analysis and Comparison Across Scenarios*

As evidenced from the previous table, the outstanding balance of all three scenarios are identical through the first milestone period ($n = 12$), since the applicable rate during the first interval is uniformly $r_1 = 0.60\%$ per month across all scenarios. Divergence start to appear at $n = 24$, where the rate differential of $\pm 0.05\%$ begins to compound. By $n = 36$, the gap between scenario A and C widened to $236,634.83 and by $n = 48$, this gap further expands to $383,841.38. This accelerating divergence is the direct result of the compounding structure in the closed-form equation of B_n. The rate differential does not accumulate linearly, but grows geometrically as the loan progresses.

From the borrower's perspective, this recurrence-based model provides a transparent, period-by-period decomposition of how each installment is split between interest and principal under each KPI scenario (successful or failing). Directly, this allows the borrower to quantify the financial value of investing in KPI-improving activities, such as accelerating capacity installation or improving operational capacity. In this case, the $228,123.86 in potential interest savings under Scenario A constitutes a concrete, mathematically fixed figure that can be weighed against the cost of sustainability initiatives.

In conclusion, the recurrence relation framework developed in this paper allows both parties, either borrowers and lenders as necessary, to compute Sustainability-Linked Loan figures at any point in time during the loan tenor, facilitating more informed and transparent contract negotiations.

ADDITIONAL MEDIA

Short presentation regarding the topic: https://youtu.be/EgQn5rnfUTQ

Google sheets link to the computation: https://docs.google.com/spreadsheets/d/12qcOtOGZu2uRfUkfswNhGi_GuaNcWVBmTuJc_PIxXiw/edit?usp=sharing

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STATEMENT

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